# Eukalyptus <br> (Koala Environment Simulator) 

## Design: Component error rates

## Revision History

| Version | When | What | Who |
| :--- | :--- | :--- | :--- |
| 0.1 | 28.04 .2004 | Initial version | Thabo Beeler |
| 0.2 | 01.05 .2004 | Corrected some error functions | Thabo Beeler |
| 0.3 | 26.06 .2004 | Added suggestions from Jörg Conradt | Jörg Conradt |

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## Overview

All the components in the real world have a systematic and/or a stochastic error. In the following we will describe the errors and the error functions.

## Drive Control

The error occurs while moving because of the physical properties of the ground. The simulator moves the koala agent not continuously but discretly at a given frequency. So every time the location changes the error applies. The error has an influence on the position and on the orientation of the koala. The error is a Gaussian function, so the formula is:

$$
\begin{array}{ll}
L_{n+1} & =L_{n}+\left[\square x_{n+1}, \square y_{n+1}\right] \\
\alpha_{n+1} & =\alpha_{n}+\arctan \left(\square y_{n+1} / \square x_{n+1}\right) \\
& \\
\square x_{n+1} & =\square x_{n} *\left(1 \pm \operatorname{random}() * \sigma_{\text {Drive }}\right) \\
\square y_{n+1} & =\square y_{n} *\left(1 \pm \operatorname{random}() * \sigma_{\text {Drive }}\right) \\
\mathrm{L} & =\text { the location of the koala } \\
\alpha & =\text { the orientation of the koala } \\
\square \mathrm{x} & =\text { the distance to move on the abscissa } \\
\square \mathrm{y} & =\text { the distance to move on the ordinate } \\
\sigma_{\text {Drive }} & =\text { the error rate }
\end{array}
$$

Of course, this error has a major impact on the path integration.

## Cameras

The cameras have multiple errors.

- Perception error = the landmark may not be seen.
- Recognition error = the landmark is recognized wrong.
- Position error
- Distance error = the distance to the landmark is calculated wrong.
- Angular error $=$ the angle under which the landmark was perceived is erroneous.


## Perception error

The perception error depends on the angle at which a landmark is perceived.


Figure 1: perception process (schematic)

It is a Boolean error rate. The result is just perceived or not perceived.
The formula to calculate the error probability is:

$$
\begin{align*}
& p_{\text {err }}=4\left(\frac{\alpha}{\alpha_{\text {max }}}\right)^{2}+\sigma_{\text {perc }}  \tag{5}\\
& \alpha \quad=\text { the angle under which the landmark is perceived } \\
& \alpha_{\text {max }}=\text { the maximal angle under which the landmark can be seen } \\
& \sigma_{\text {perc }}=\text { the error rate at } 0 \text { degrees. This can be read as the chance to miss the landmark at } 0^{\circ} \text {. }
\end{align*}
$$

The following chart shows the error function for a typical landmark with $\alpha_{\max }=120^{\circ}$ and $\sigma_{\text {perc }}=0.01$.

Perception error ( $\mathbf{p}=\mathbf{0 . 0 1}$ )


Figure 2: Perception error for $p=0.01$

The next chart shows multiple angles with $\sigma_{\text {perc }}=0.01$. It shows that a view angle of $360^{\circ}$ does not mean, that the landmark can be perceived equal well from all sides. The landmark still has an orientation. A landmark, which can be perceived from all sides equally well does not have such an error distribution, because it does not depend on the angle it is perceived. Actually such a landmark consists of infinitely many landmarks, one at each possible orientation. We therefore need to circumvent the error function. This can easily be done by setting $\alpha_{\text {max }}$ to a very high number. In the example we used $10000^{\circ}$ which results in an equally distributed error rate.


Figure 3: Perception error for $p=0.1$

But the perceived error depends also on the distance to the landmark. The further away the landmark, the smaller it is, and the bigger is the perception error. So we need a function that grows with increasing distance and reaches its maximum (1) at the maximum sight distance $\mathrm{d}_{\text {max }}$.
On the other hand, when the landmark is too close, it cannot be perceived as well. We will approximate the error, when the landmark is too close, by applying a minimum sight distance $\mathrm{d}_{\text {min }}$.

The chosen formula is exponential at base 2 .

$$
\begin{align*}
& p=2^{\left(d-d_{\max }\right)}  \tag{6}\\
& \quad \begin{array}{l}
\mathrm{d} \quad \\
\begin{array}{l}
\text { distance to the landmark } \\
\mathrm{d}_{\text {max }}
\end{array} \\
=\text { maximum sight distance }
\end{array}
\end{align*}
$$

Distance error


Figure 4: Distance errors for different bases

Because we calculate the distance with the theorem of Pythagoras we get the squared distance. To prevent a time-consuming square root calculation, we could approximate the original formula by the following:

$$
\begin{align*}
& p=2^{\left(\frac{\left(d^{2}-d_{\max }^{2}\right)}{2 d_{\max }}\right)}  \tag{7}\\
& \begin{array}{l}
\mathrm{d} \\
\mathrm{~d}_{\max } \quad=\text { distance to the landmark } \\
=\text { maximum sight distance }
\end{array}
\end{align*}
$$

The approximation works pretty well.


Figure 5: Approximated distance errors for different bases
The final perception error rate is therefore a combination of these two functions and depends on the distance and the angle at which a landmark is perceived. The final formula is:

$$
\begin{equation*}
p_{\text {err }}=4\left(\frac{\alpha}{\alpha_{\max }}\right)^{2}+\sigma_{\text {perc }}+2^{\left(d-d_{\max }\right)} \tag{8}
\end{equation*}
$$

As one can see, this formula has three dimensions:


Figure 6: Perception error function ( $x=$ perceived distance, $y=$ perceived angle, $z=$ perception error $)$

## Recognition error

The recognition error again depends on the perception angle and the distance. So we get the same distribution. The difference is that when a landmark was not correctly recognized it returns a wrong landmark id. The probability that a random landmark $L_{i}$ will be returned is:

$$
\begin{equation*}
p_{L i}=\frac{1}{n} \tag{9}
\end{equation*}
$$

$\mathrm{n}=$ the amount of landmarks in the world

## Positional error

## Distance error

The stereoscopic sight of the koala is not very good, so distance estimation is only accurate for near landmarks.

$$
\begin{array}{ll}
d_{\text {est }} & =d_{\text {real }}+\operatorname{random}() * \square_{\text {far }}+\operatorname{random}() * \square_{\text {near }} \\
\square_{\text {far }}= & \sigma_{\text {dist }} d_{\text {mid }} 2\left(\frac{\left(d_{\text {real }}-d_{\text {mid }}\right)}{2}\right) \\
\square_{\text {near }}= & -\sigma_{\text {dist }} d_{\text {real }}  \tag{12}\\
& =\text { the medium sight distance. At this distance, the error is } \pm \sigma_{\text {dist }} \\
d_{\text {mid }} & \quad=\text { the real distance } \\
d_{\text {real }} & =\text { the estimated distance }
\end{array}
$$

This error distribution has the effect that the estimated mean distance is close to the real distance for near landmarks.

Distance estimation error


Figure 7: Distance error ( short distances)

For far distances the estimated mean distance is highly erroneous.

Distance estimation error


Figure 8: Distance error (long distances)

## Angular error

This again is a Gaussian error. We have got two cameras and both have such an error. For the simulator though, we approximate it with one single camera positioned between the two others. The approximation works fine and we cut the computational effort half. The koala also reports just one angle to the sensory cortex.


Figure 9: Angular error (schematic)

So we are interested in $\alpha$ only. The calculation is fairly simple:

$$
\begin{array}{ll}
\alpha= & \frac{\alpha_{L}+\beta_{L}+\alpha_{R}+\beta_{R}}{2}  \tag{13}\\
\alpha \quad \quad=\text { perceived angle relative to the koala } \\
\alpha & =\text { orientation of the camera relative to the koala } \\
\alpha_{\text {LRR }} & =\text { perceived angle relative to the camera }
\end{array}
$$

Of course every angle would have a separate and independent Gaussian error. But again, we approximate them with one single error.
$\alpha_{\text {est }}=\alpha *\left(1 \pm \operatorname{random}() * \sigma_{\text {ang }}\right)$
$\alpha \quad=$ perceived angle relative to the koala
$\alpha_{\text {est }} \quad=$ the estimated perception angle relative to the koala
$\sigma_{\mathrm{ang}} \quad=$ the angular error of the camera module

## Gyroscope

The error of the gyroscope is assumed to be a Gaussian error integrated over time.
$\alpha_{n+1}=\alpha_{n}+\left(1 \pm \operatorname{random}() * \sigma_{\text {Gyroscope }}\right) * \square_{\alpha t}$
$\alpha \quad=$ the estimated angle of the gyroscope
$\sigma_{\text {Gyroscope }}=$ the error of the gyroscope
$\square_{\alpha t} \quad=$ the relative rotation the koala did at time t

## Magnetic compass

The error of the magnetic compass is somehow special. The error results from interferences from magnetic fields. The power of a magnetic field vanishes inverse proportional to the squared distance.

$$
\begin{align*}
& F=\frac{\xi}{d_{M K}{ }^{2\left(L_{M}-L_{K}\right)}}  \tag{16}\\
& \xi \quad=\text { power of the field in the center } \\
& \begin{array}{l}
\mathrm{d}_{\text {MK }} \quad \text { the distance between the center of the magnetic field and the koala } \\
\mathrm{L}_{\mathrm{M}} \quad=\text { the elocation of the center of the magnetic field } \\
\mathrm{L}_{\mathrm{K}} \quad=\text { the location of the koala }
\end{array}
\end{align*}
$$

So if we have n magnetic fields that interfere, we get a resulting force vector with this formula:

$$
\begin{align*}
\mathrm{F}_{\mathrm{tot}} & =\Sigma\left(\mathrm{F}_{\mathrm{i}}\right)  \tag{17}\\
& =\Sigma\left(\frac{\xi_{i}}{d_{M K_{i}}^{2\left(L_{M_{i}}-L_{K}\right)}}\right) \tag{18}
\end{align*}
$$

Note: The influence of the north pole is just one of these force vectors, lets assume $\mathrm{F}_{0}$.

